38/11 $f(x) = \frac{1}{2+x}$; to show $\frac{1}{3+\Delta x} \approx \frac{1}{3} - \frac{1}{9}\Delta x$

choose $a = 1$

↓ not an x

$f(x) \approx f(a) + f'(a)(x-a)$

$f(x) \approx f(1) + f'(1)(x-1)$

eqn of tangent line: $f(x) \approx \frac{1}{3} - \frac{1}{9}(x-1)$

$f(1) = \frac{1}{2+1} = \frac{1}{3}$

$f'(x) = \frac{d}{dx}(2+x)^{-1} = -(2+x)^{-2}(1) = \frac{-1}{(2+x)^2}$

$f'(1) = \frac{-1}{(2+1)^2} = -\frac{1}{9}$

Q: for what xs (← is an x) will this be a Good approximation

A: xs close to 1

Idea: instead of x, use $1+\Delta x$

When I see x, replace with $(1+\Delta x)$

$f(1+\Delta x) \approx \frac{1}{3} - \frac{1}{9}((1+\Delta x)-1)$

$f(x) = \frac{1}{2+x}$

$f(1+\Delta x) = \frac{1}{2+(1+\Delta x)} = \frac{1}{3+\Delta x}$

$\frac{1}{3+\Delta x} \approx \frac{1}{3} - \frac{1}{9}(\Delta x)$

(6.7/12a) $v(t)=2t-4$; $[0,6]$

find distance & displacement

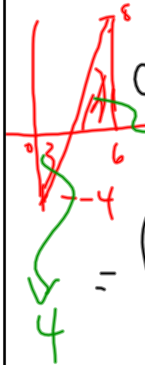
displacement $= \int_0^6 2t-4\,dt$

$= (t^2-4t)\Big|_0^6 = (36-24)-(0)$

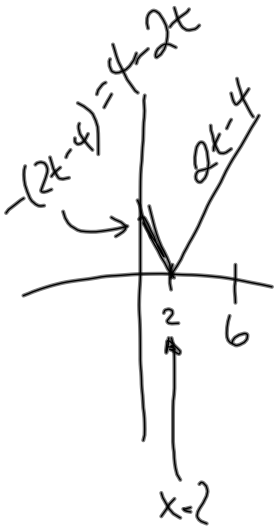
$= 12$

2

2



distance $= \int_0^6 |2t-4|\,dt$



$= \int_0^2 4-2t\,dt + \int_2^6 2t-4\,dt$

$= (4t-t^2)\Big|_0^2 + (t^2-4t)\Big|_2^6$

$= (8-4)-(0)+(36-24)-(4-8)$

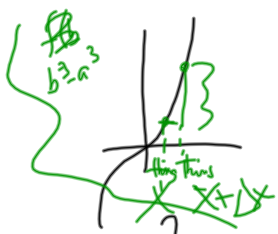
$= (4)+(12)-(-4) = 20$

4 | 16

38/33 $y = x^3$

find formulas for Δy and dy.

$$\Delta y = f(x+\Delta x) - f(x)$$
$$= (x+\Delta x)^3 - (x)^3$$
$$= \left[x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3\right] - (x)^3$$
$$\Delta y = 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3$$



$(a+b)^3 =$
$(a+b)(a+b)(a+b)$
$(a^2+2ab+b^2)(a+b)$
$a^3+2a^2b+ab^2$
$+ \quad a^2b+2ab^2+b^3$
$a^3+3a^2b+3ab^2+b^3$

$(a+b)^3$

$1a^3$
$3a^2b$
$3ab^2$
$1b^3$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

$(x+1)^6$ Binomial Theorem

$$x^6+6x^5+15x^4+20x^3+15x^2+6x+1$$

$y = x^3$
$dy = 3x^2\,dx$

5.4 Rectilinear Motion
(the derivative view)

⇒ when is a particle speeding up?
A: when acceleration and velocity "pull in same direction"

5.5 Absolute Minimum or Maximum

⇒ either a relative minimum or maximum
OR an end point.

5.6 Applied Maximum/Minimum

$\Rightarrow$ * find an equation in 1 variable
* differentiate wrt that variable
* find $\overset{abs}{\text{max}}$ or min

5.8 Rolle's Theorem/Mean Value Theorem

$\Rightarrow$ Rolle's Theorem: $f(a)=f(b)$ ... Then MVT

$\Rightarrow$ MVT: $\exists c \in (a,b)$ with

$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

there exists a c ($x=c$)
in the interval (a,b)
with $$f'(c)=\frac{f(b)-f(a)}{b-a}$$

Jan 12-8:33 AM

3.8/19 estimate $(3.02)^4$ using local linear approx.

$f(3) = 3^4 = 81$

$f(x) = x^4$

$f'(x) = 4x^3$

$f'(3) = 4 \cdot 3^3 = 4 \cdot 27 = 108$

$(3.02)^4 \approx 3^4$

$f(x) \approx f(a) + f'(a)(x-a)$

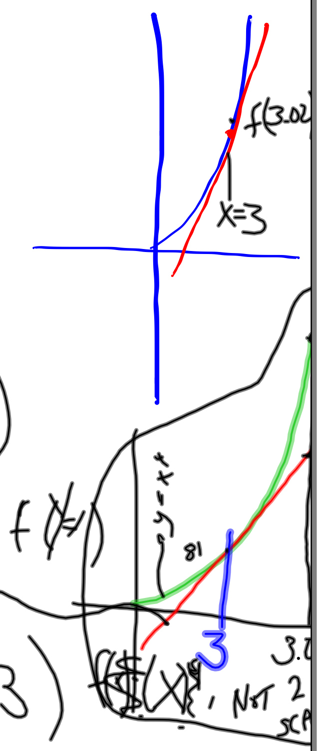
$f(x) \approx f(3) + f'(3)(x-3)$

$f(x) \approx 81 + 108(x-3)$

$f(3.02) \approx 81 + 108(3.02-3)$

$= 81 + 108(.02) = 81 + 2.16$

≈ 83.16

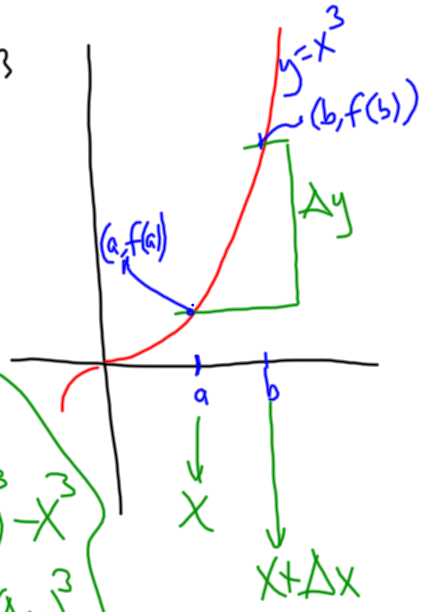


$3.02^4 = 83.1816\ldots$

3.8/33 $y = x^3$ find formulas dy and Δy

Δy = chg in y of $y = x^3$

$\Delta y = f(b) - f(a) = b^3 - a^3$



$$\Delta y = f(x+\Delta x) - f(x)$$
$$\Delta y = (x+\Delta x)^3 - (x^3)$$
$$\Delta y = x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - x^3$$
$$\Delta y = 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3$$

$$dy = 3x^2\,dx \qquad \frac{dy}{dx} = \text{derivative}$$

$$(a+b)^3 = \underbrace{(a+b)(a+b)}(a+b)$$
$$(a^2+2ab+b^2)(a+b)$$
$$= a^3 + 2a^2b + ab^2$$
$$+\ a^2b + 2ab^2 + b^3$$
$$a^3 + 3a^2b + 3ab^2 + b^3$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
$\boxed{1\ \ 5\ \ 10\ \ 10\ \ 5\ \ 1}$

$(x+1)^5$